Math 3 **Name** ____________________

# 8-1 What are Exponential and Logarithmic Functions

**Class** ________

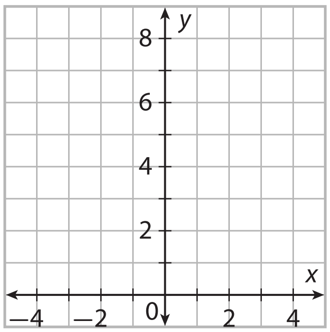
An **exponential function** is a function of the form $f(x) = b^x$, where the base $b$ is a positive constant other than 1 and the exponent $x$ is a variable. Notice that there is no single parent exponential function because each choice of the base $b$ determines a different function.

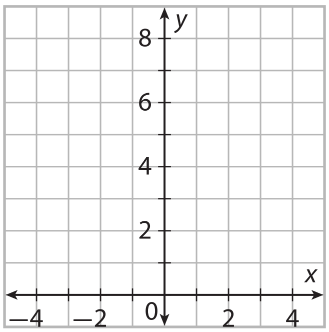
Ⓐ Complete the input-output table for each of the parent exponential functions below.

| $x$ | $f(x) = 2^x$ |
|---|---|
| −3 | |
| −2 | |
| −1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |

| $x$ | $p(x) = 10^x$ |
|---|---|
| −3 | |
| −2 | |
| −1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |

Ⓑ Graph the parent functions $f(x) = 2^x$ and $p(x) = 10^x$ by plotting points.




Ⓒ What is the domain of each function?

Domain of $f(x) = 2^x$: $\{x|$ ____________________ $\}$

Domain of $p(x) = 10^x$: $\{x|$ ____________________ $\}$

Ⓓ What is the range of each function?

Range of $f(x) = 2^x$: $\{y|$ ____________________ $\}$

Range of $p(x) = 10^x$: $\{y|$ ____________________ $\}$

Ⓔ What is the $y$-intercept of each function?

$y$-intercept of $f(x) = 2^x$: ☐

$y$-intercept of $p(x) = 10^x$: ☐

Ⓕ What is the trend of each function?

In both $f(x) = 2^x$ and $p(x) = 10^x$, as the value of $x$ increases, the value of $y$ increases/decreases.

## Reflect

**1.** Will the domain be the same for every exponential function? Why or why not?

**2.** Will the range be the same for every exponential function in the form $f(x) = b^x$, where $b$ is a positive constant? Why or why not?

**3.** Will the value of the $y$-intercept be the same for every exponential function? Why or why not?

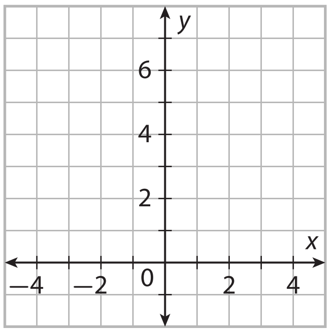
Exponential functions with bases between 0 and 1 can be transformed in a manner similar to exponential functions with bases greater than 1. Begin by plotting the parent functions of two of the more commonly used bases: $\frac{1}{2}$ and $\frac{1}{10}$.

(A) To begin, fill in the table in order to find points along the function $f(x) = \left(\frac{1}{2}\right)^x$. You may need to review the rules of the properties of exponents, including negative exponents.

| $x$ | $f(x) = \left(\frac{1}{2}\right)^x$ |
|---|---|
| −3 | 8 |
| −2 | |
| −1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |

(B) What does the end behavior of this function appear to be as $x$ increases?

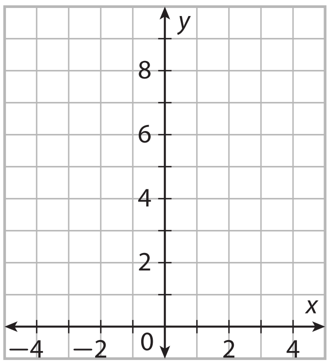
(C) Plot the points on the graph and draw a smooth curve through them.




(D) Complete the table for $f(x) = \left(\frac{1}{10}\right)x$.

| $x$ | $f(x) = \left(\frac{1}{10}\right)^x$ |
|---|---|
| −3 | 1000 |
| −2 | |
| −1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |

(E) Plot the points on the graph and draw a smooth curve through them.



(F) Fill in the following table of properties:

| | $f(x) = \left(\frac{1}{2}\right)^x$ | $f(x) = \left(\frac{1}{10}\right)^x$ |
|---|---|---|
| Domain | $\{x \mid -\infty < x < \infty\}$ | $\{x \mid \square\}$ |
| Range | $\{y \mid \square\}$ | $\{y \mid \square\}$ |
| End behavior as $x \to \infty$ | $f(x) \to \square$ | $f(x) \to \square$ |
| End behavior as $x \to -\infty$ | $f(x) \to \square$ | $f(x) \to \square$ |
| $y$-intercept | $\square$ $\square$ | $\square$ $\square$ |

(G) Both of these functions [decrease/increase] throughout the domain.

(H) Of the two functions, $f(x) = \left(\frac{1}{\square}\right)^x$ decreases faster.

**Reflect**

4. **Make a Conjecture** Look at the table of properties for the functions. What do you notice? Make a conjecture about these properties for exponential functions of the form $f(x) = \left(\frac{1}{n}\right)^x$, where $n$ is a constant.

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5. **Make a Conjecture** What is the difference between the graphs on Page 1 and the Graph on Page 2?

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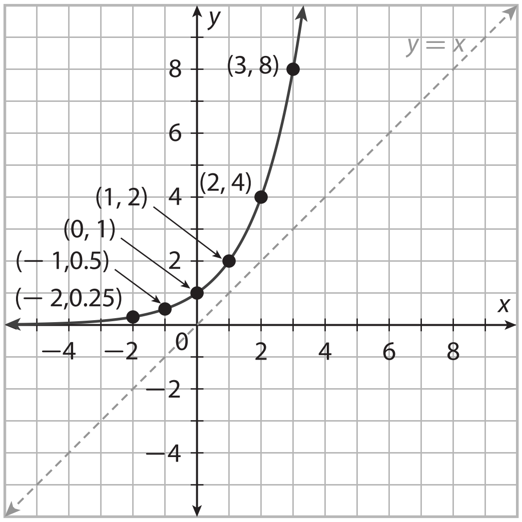
## Explore — Understanding Logarithmic Functions as Inverse of Exponential Functions

An exponential function such as $f(x) = 2^x$ accepts values of the exponent as inputs and delivers the corresponding power of 2 as the outputs. The inverse of an exponential function is called a **logarithmic function**. For $f(x) = 2^x$, the inverse function is written $f^{-1}(x) = \log_2 x$, which is read either as "the logarithm with base 2 of $x$" or simply as "log base 2 of $x$." It accepts powers of 2 as inputs and delivers the corresponding exponents as outputs.

(A) Graph $f^{-1}(x) = \log_2 x$ using the graph of $f(x) = 2^x$ shown. Begin by reflecting the labeled points on the graph of $f(x) = 2^x$ across the line $y = x$ and labeling the reflected points with their coordinates. Then draw a smooth curve through the reflected points.




(B) Using the labeled points on the graph of $f^{-1}(x)$, complete the following statements.

$f^{-1}(0.25) = \log_2$ ☐ = ☐

$f^{-1}(0.5) = \log_2$ ☐ = ☐

$f^{-1}(1) = \log_2$ ☐ = ☐

$f^{-1}(2) = \log_2$ ☐ = ☐

$f^{-1}(4) = \log_2$ ☐ = ☐

$f^{-1}(8) = \log_2$ ☐ = ☐

**6.** Explain why the domain of $f(x) = 2^x$ doesn't need to be restricted in order for its inverse to be a function.

**7.** State the domain and range of $f^{-1}(x) = \log_2 x$ using set notation.

**8.** Identify any intercepts and asymptotes for the graph of $f^{-1}(x) = \log_2 x$.

**9.** Is $f^{-1}(x) = \log_2 x$ an increasing function or a decreasing function?

**10.** How does $f^{-1}(x) = \log_2 x$ behave as $x$ increases without bound? As $x$ decreases toward 0?